begun per second is, in the case supposed, $B = P + F = n \div \tau + \frac{1}{2} B$, and so $\frac{1}{2} B = n \div \tau$. Hence $(350 \div \theta)\, n\, g = n \div \tau$, or $g = (\theta \div 350\tau)$. But when $F = \frac{1}{2} B$, $(\theta \div \tau) = \log_e 2$, and so $g = 0.00\,2$ nearly.

[6] Hall, E. H., *Boston, Proc. Amer. Acad. Arts and Sci.*, **50,** 1914, (67–103).

THE GRAVIMETRIC SURVEY OF THE UNITED STATES

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The gravimetric survey of the United States really began in 1890 with the introduction of the Mendenhall one-half second invariable pendulum. Previous to that date, 13 stations had been established but in that older work pendulums were used which gave inaccurate results, as later work showed. We shall not, therefore, consider the gravity results obtained before the use of the Mendenhall pendulum.

This pendulum consists of a bob and stem with a suitable head into which is set an agate plane which rests on a knife edge of the same material fastened to the pendulum case. The various parts of the apparatus are illustrated and described in reports of the Coast and Geodetic Survey.[1] The Mendenhall pendulums are used to determine the difference in the intensity of gravity at two stations.

The probable error of the value of gravity at Washington, determined from Potsdam by the relative method, is ± 0.001 dyne. The probable error of a station in the United States, other than that at Washington, is about ± 0.002 dyne. This is about one part in one-half million. The error of the absolute value at Potsdam enters all other values of gravity based upon the Potsdam system.

Between the years 1891 and 1907, 47 stations were established in the United States, while since January, 1909, 212 additional stations have been established, making 259 in all. The Coast and Goedetic Survey has planned to continue its gravimetric survey for an indefinite period, with a view to covering the large areas now lacking in stations, and also local areas where there are special problems to be investigated.

There are two immediate purposes to be served in carrying on this work. First, to collect data from which more accurate values may be obtained for the flattening of the earth and for the terms in the gravity formula. Second, to obtain values of the intensity of gravity at laboratories as these values are needed in certain physical and chemical work.

Another important use to which the gravimetric survey may be put is in researches into the subject of isostasy. While an old subject[2] isostasy has only recently become a vital matter to be considered in

most geophysical problems. The work done in the Survey[3,4,5] shows that, at some depth below sea level (of the order of 100 kilometers) the pressure of any unit column is very nearly equal to that of any other unit column. For instance, the pressure exerted on a square mile of the imaginary surface at a depth of say 96 kilometers[6] below sea level at the sea coast or under the plains is about the same as the pressure on an equal area at the same depth below sea level under the Rocky Mountains or under any other mountain masses.

If we assume that the equalization of pressures at the supposed depth, called the depth of compensation, is perfect, then we must conclude that the land masses are counterbalanced by deficiencies in density of the materials below sea level, under the topographic features. It is not probable that the pressures are exactly equal for small areas of the surface at the depth of compensation. It is very probable that they are practically equal for areas of the order of ten thousand square miles. It is one of the important problems of the geodesist to collect sufficient data to show the minimum area which may be in a high state of isostasy. Another problem for him to investigate is the distribution horizontally and vertically of the deficiencies in mass, which balance the material which is above sea level.

For the purposes of making the computations, the compensating deficiencies of density or mass are supposed to be uniformly distributed directly under the topographic feature from the earth's surface to the depth of compensation. And it is also assumed that the negative masses exactly equal the positive masses which are above sea level.

We cannot tell from the data now at hand just how near the truth are these assumed conditions. But we do know that they are very much nearer the truth than those conditions based upon a rigid earth, with the topographic features held up by the strength of the earth's outer material. The assumed isostatic conditions are also shown to be much nearer the truth than those based upon the theory that the topographic features should be ignored.

It will be interesting to consider briefly the results of the latest investigations by the Coast and Geodetic Survey upon the subject of gravity and isostasy.[5]

We may assume that the method of reduction which shows the smallest effect of systematic or constant errors is the nearest the truth. For instance, there may be used in the tests five classes of topography (the sea being ignored, as we have no very accurate gravity observations at sea). They are indicated in the tables below. The isostatic or Hayford method of reduction was used with two depths of compensa-

tion and with two different gravity formulas; otherwise they are identical. The Bouguer reduction postulates a highly rigid earth and the free air reduction an earth with no rigidity.

RELATION BETWEEN THE GRAVITY ANOMALIES AND THE TOPOGRAPHY

Mean anomalies

With regard to sign

	NUMBER OF STATIONS	MEAN ANOMALY			
		Hayford, 1912; depth 113.7 km.	Hayford, 1916; depth 60 km.	Bouguer	In free air
Coast stations	27	−0.009	−0.003	+0.017	+0.017
Stations near coast	46	−0.001	+0.002	+0.004	+0.017
Stations in interior, not in mountainous regions	88	−0.001	−0.001	−0.028	+0.009
Stations in mountainous regions					
Below the general level	36	−0.003	0.000	−0.107	−0.008
Above the general level	20	+0.001	+0.016	−0.110	+0.058
All stations (except the two Seattle stations)	217	−0.002	+0.001	−0.036	+0.013

Without regard to sign

	NUMBER OF STATIONS	Hayford, 1912; depth 113.7 km.	Hayford, 1916; depth 60 km.	Bouguer	In free air
Coast stations	27	0.018	0.012	0.021	0.022
Stations near coast	46	0.021	0.020	0.025	0.023
Stations in interior, not in mountainous regions	88	0.019	0.019	0.033	0.020
Stations in mountainous regions					
Below the general level	36	0.020	0.018	0.108	0.024
Above the general level	20	0.017	0.022	0.111	0.059
All stations (except the two Seattle stations)	217	0.019	0.019	0.049	0.025

The table above shows that when the reductions are made by either of the Hayford methods the range of the mean anomalies with regard to sign is very nearly zero in most cases. The Bouguer and free air anomalies are much larger than the isostatic anomalies. By anomaly is meant the difference between the observed and the computed values of gravity at a station.

The table indicates strongly that the conditions under which the isostatic or Hayford reductions were made are very close to the truth. The evidence is that the depth 113.7 kilometers is closer to the truth than 60 kilometers, for in the former case the mean anomaly for the classes of topography indicated above varies from +0.001 to −0.009 dyne, while with the latter the range is from +0.016 to −0.003. The

Bouguer range is from +0.017 to −0.110 dyne. The free air means vary from +0.058 to −0.008.

It is important to consider whether the compensation occurs directly under a topographic feature, a mountain mass for instance, or is distributed through a column having a cross section somewhat greater in area than the base of the feature.

The compensation was distributed in horizontal extent to distances of 19, 59 and 167 kilometers in all directions from the stations. These distances correspond to the outer limits of zones K, M and O, which were used in computing the topographic and compensation corrections. It was found that the mean anomaly with and without regard to sign was approximately the same for each method of distribution horizontally and for local distribution, if all stations were treated as a single group. Consequently, no one method seemed to be more probable than any other. The stations were next considered in five groups, according to the topography, with the results shown in the following table.

RELATION OF LOCAL-COMPENSATION ANOMALIES AND REGIONAL-COMPENSATION ANOMALIES TO TOPOGRAPHY

	ANOMALY LOCAL COMPENSATION	ANOMALY. REGIONAL-COMPENSATION WITHIN OUTER LIMIT OF		
		Zone K 19 km.	Zone M 59 km.	Zone O 167 km.
At 18 Coast Stations				
Mean with regard to sign	−0.004	−0.004	−0.004	−0.006
Mean without regard to sign	0.018	0.018	0.018	0.020
At 25 stations near coast				
Mean with regard to sign	−0.002	−0.001	−0.001	−0.001
Mean without regard to sign	0.022	0.021	0.021	0.022
At 39 interior stations, not in mountains				
Mean with regard to sign	+0.001	+0.002	+0.002	+0.003
Mean without regard to sign	0.017	0.018	0.018	0.017
At 22 mountain stations below general level				
Mean with regard to sign	0.000	+0.001	+0.003	+0.006
Mean without regard to sign	0.017	0.017	0.018	0.019
At 18 mountain stations above general level				
Mean with regard to sign	+0.003	+0.003	0.000	−0.010
Mean without regard to sign	0.018	0.018	0.017	0.020

The mean anomaly without regard to sign is not decidedly in favor of either method. But the means with regard to sign seem to point strongly against the distribution of the compensation out to a distance of 167 kilometers from the stations, for there we have a range in the mean anomalies of 0.016 dyne between the 22 stations in mountainous

regions below the general level and the 18 mountain stations above the general level. The range for local and the other two methods is, in each case, only 0.007 dyne.

We must conclude that the regional distribution out to 59 kilometers is as likely to be true as the local distribution of the compensation. There may be some distance beyond 59 kilometers and less than 167 kilometers which would show a smaller range in the mean anomalies than any of the distributions considered above. It seems to be reasonable to expect that the distance in question is nearer 59 than 167 kilometers.

Tests made to show which is the most probable depth of compensation indicated that when the stations were taken as a single group the value is about 60 kilometers. But when the stations were arranged according to the topography the depth of 95 kilometers is the most probable depth. The depths of compensation, determined from deflections of the vertical,[3] are from the first investigation 113.7 kilometers and from the later one, 122 kilometers. When the deflection stations in mountain regions only were considered the depth is 97 kilometers. The mean of this and the depth from gravity stations is 96 kilometers. It is believed this is the best value from all geodetic data.

It is improbable that the compensation is distributed uniformly from sea level to the depth of compensation. It is not probable that what may be considered the normal distribution of densities of the material in the outer portions of the earth obtains under all places on the earth's surface.

The present data are not sufficient to enable one to compute the actual distribution of densities under any given area, but there are reasons to believe that approximations to the actual distribution may be made.

In a study of the relation between the gravity anomalies and the geological formations, as indicated by surface material, it developed that at stations on the pre-Cambrian areas gravity is nearly always in excess. It is known that the older rocks have greater densities than normal and as this material is close to the gravity stations there should result a greater intensity of gravity. The excess of gravity at a station may give some idea of the depth of this older rock.

The stations on the Cenozoic formations have a tendency to a deficiency in gravity. This seems to be logical as the rocks in this formation have densities less than normal. Here again the size of the deficiency may indicate the approximate depth of the Cenozoic formation. The Paleozoic stations tend to be negative and the Mesozoic stations

positive, but there is no such evident relation between the densities and the gravity as with the two other formations. The average densities of the Paleozoic and Mesozoic rocks are about equal.[7]

There is given below a table which shows that relations exist between the gravity anomalies and the geological formations in the United States. A positive anomaly indicates an excess in gravity and a negative anomaly a deficiency.

RELATION BETWEEN THE GRAVITY ANOMALIES AND THE GEOLOGIC FORMATION

GEOLOGIC FORMATION	NUMBER OF STATIONS			MEAN ANOMALY	
	With plus anomalies	With minus anomalies	All	With regard to sign	Without regard to sign
Pre-Cambrian	12	2	14	+0.019	0.023
Paleozoic	23	49	72	−0.011	0.021
Mesozoic	25	11	36	+0.009	0.017
Cenozoic	22	32	55	−0.007	0.019

A study was made of the stations in India, 73 in all, and these showed that gravity at Cenozoic stations is, in general, too small. There were very few stations in the other formations which made it impossible to draw any definite conclusions from them.

RELATION BETWEEN THE GRAVITY ANOMALIES AND THE GEOLOGIC FORMATION FOR STATIONS IN INDIA

GEOLOGIC FORMATION	NUMBER OF STATIONS			MEAN ANOMALY	
	With plus anomalies	With minus anomalies	All	With regard to sign	Without regard to sign
Pre-Cambrian	6	2	8	+0.002	0.025
Paleozoic	2	3	6*	0.000	0.009
Mesozoic	1	0	1	+0.022	0.022
Cenozoic	11	20	31	−0.017	0.028

* One anomaly is zero.

The larger part of most of the larger anomalies may de due to deficiencies or excesses in the densities of the materials below sea level near the stations, and these deviations from normal may be compensated in the lower strata. If this is true, isostasy is more nearly perfect than has generally been supposed.

There are 42 stations in Canada which were studied but their gravity values showed no such relations to the geological formations as were found in the United States and in India.

From the values of gravity at 358 stations in the United States, Can-

ada, India and in Europe, which had been reduced by the isostatic or Hayford method, the best gravity formula deduced is

$$\gamma_0 = 978.039 \ (1 + 0.005294 \sin^2 \phi - 0.000007 \sin^2 2 \phi)$$

in which γ_0 is the value of gravity in dynes at sea level, at the latitude, ϕ, and the first term, 978.039, is the value of gravity at the equator.

From the constant 0.005294 a reciprocal of the flattening of the earth of 297.4 was derived.

The well known formula

$$C_h = -0.0003086 \, H$$

was used to correct the value of gravity for the distance above sea level. H is the elevation of the station in meters.

If we assume that the best known value of the equatorial radius of the earth is 6,378,388 meters,[8] then the polar semi-diameter is 6,356,941 meters. The difference is 21,447 meters or 13.3 miles.

The results of the investigations of the Coast and Geodetic Survey make it possible to compute the value of gravity for stations in the United States, and possibly also in any other country, with an average uncertainty in the result of about 0.020 dyne or one part in 50,000.

Further work on the gravimetric survey of the United States will enable us to obtain better values of the shape of the earth, and for the constants of the gravity formula, and will no doubt lead to important discoveries regarding the distributions of densities in the outer portions of the earth and especially within the outer ten miles.

[1] *Washington, D. C., U. S. Commerce Dept. Coast & Geod. Surv., Rep.*, 1891, Appendix No. 15; *Ibid.*, 1893, Appendix No. 12; *Ibid.*, 1910, Appendix No. 6.

[2] *London, Phil. Trans. R. Soc.*, **149**, 1859, (745); Dutton, C. E., *Washington, D. C., Bull. Wash. Phil. Soc.*, **11**, 1889, (51–64).

[3] Hayford, J. F., *U. S. Commerce Dept. Coast & Geod. Surv.*, The figure of the earth and isostasy from measurements in the United States, 1909; Supplementary investigation in 1909 of the figure of the earth and isostasy, 1909.

[4] Hayford, J. F., and Bowie, W., *U. S. Commerce Dept. Coast & Geod. Surv., Sp. Pub.*, No. 10, 1912; Bowie, W., *Ibid.*, No. 12, 1912.

[5] Bowie, W., *Ibid.*, No. 40, 1916.

[6] This, according to the latest investigation, is the most probable depth (see [5]).

[7] Barrell, J., *Chicago, Ill., J. Geol. Univ. Chic.*, **22**, 1914, (215).

[8] See Hayford, loc. cit., Note 3, 1909, (54).